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Temperature Distributions in Channel Walls with Translucent Thermal Barrier Coatings

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Thermal barrier coatings are important for reducing metal temperatures and thermal gradients in high-temperature applications. At elevated temperatures radiative transfer becomes important, and if several hot surfaces view each other, the coated surfaces exchange radiation. Usually in radiative exchange computations, the surfaces are opaque and each has a unique radiating temperature. Some thermal barrier coating materials such as zirconia are partially transparent to thermal radiation. Hence, an exchange involving coated walls is complicated by radiation penetrating into and coming out of each coating. Radiation leaving an area depends on the temperature distribution inside its coating that is unknown and is affected by the exchange process; each area does not have a unique temperature. The analysis here is for a hot transparent gas flowing in a symmetric parallel-plate channel with translucent coatings on the inside. The coatings view each other and exchange radiation. From symmetry the flux leaving each coating equals its incident value. Relations are developed for the temperature distribution within each coating that satisfies zero-net radiation at the coating surface. This differs from having cold surroundings. In that case the net radiation is not zero because there is radiation from the coating to the surroundings.

Nomenclature

a_ν, a = spectral or gray absorption coefficient of translucent coating, m^{-1}
c_0 = velocity of electromagnetic propagation in vacuum, m/s
$e_{\nu b}\,d\nu$ = blackbody spectral emission in vacuum, W/m^2; $\tilde{e}_{\nu b} = e_{\nu b}/\sigma T_{g1}^4$
F_j = fraction of blackbody energy from $\nu = 0$ to a band frequency limit
G_ν = spectral flux quantity in two-flux method, W/m^2
$\tilde{G}_\nu$ = dimensionless quantity, $G_\nu/\sigma T_{g1}^4$
g_ν = Green's function for $\tilde{G}_\nu(X)$ in coating
H = dimensionless convective heat transfer coefficient, $h/\sigma T_{g1}^3$
h_1, h_2 = convective heat transfer coefficients at boundaries, W/m^2 K
k_c, k_m = thermal conductivity of coating, and metal or opaque substrate, W/m · K
m_ν = quantity $(a_\nu + \sigma_{s\nu})\delta_c[3(1 - \Omega_\nu)]^{1/2}$
N_c, N_w = conduction–radiation parameters, $k_c/\sigma T_{g1}^3\delta_c$, $k_m/\sigma T_{g1}^3\delta_m$
n = refractive index of translucent coating
$P_{m\nu}$ = quantity $2(2 - \varepsilon_{m1})/[3(a_\nu + \sigma_{s\nu})\delta_c\varepsilon_{m1}]$
q = heat flux, W/m^2
$\tilde{q}$ = dimensionless heat flux, $q/\sigma T_{g1}^4$
q_{r1}, q_{r2} = externally incident radiative fluxes, Fig. 1, W/m^2
q_{tot} = total heat flux by combined conduction and radiation, W/m^2
$q_{\nu r}\,d\nu$ = spectral radiative heat flux, W/m^2
T = absolute temperature, K
T_{g1}, T_{g2} = gas temperature on inside and outside of channel, Fig. 1, K
T_{s2} = temperature of blackbody surroundings on outside of channel, Fig. 1, K
t = dimensionless temperature, T/T_{g1}
X = dimensionless coordinate, x/δ_c
x = coordinate in coating and metal combination, Fig. 1, m
δ_c, δ_m = thicknesses of coating, and metal or opaque substrate, m
ε_{m1}, ε_{m2} = emissivity at inside and outside of metal or opaque substrate, Fig. 1
κ_c = optical thickness of coating, $(a + \sigma_s)\delta_c$
κ_j = coating optical thickness in jth frequency band, $(a_j + \sigma_{sj})\delta_c$
ν = frequency of radiation, 1/s
ξ = dimensionless integration variable in the X direction
ρ_i, ρ_o = diffuse reflectivities at inside and outside interfaces of exposed surface of coating, Fig. 1
σ = Stefan–Boltzmann constant, W/m^2 K^4
$\sigma_{s\nu}$, σ_s = spectral or gray scattering coefficient in semitransparent coating, m^{-1}
Ω_ν = scattering albedo, $\sigma_{s\nu}/(a_\nu + \sigma_{s\nu})$

Subscripts

c, m = coating, and metal or opaque substrate
h = homogeneous solution of two-flux equation
j = jth frequency band
l, u = lower and upper limits of a frequency band
r = radiative
ν = frequency dependent

Introduction

THERMAL barrier coatings can be used for some high-temperature applications to protect metal parts from a high-temperature environment and reduce metal temperatures and thermal gradients. At elevated temperatures radiative transfer may be important, and if there are several coated surfaces that view each other, the surfaces will exchange radiation. In conventional radiative exchange calculations the radiating surfaces are opaque, and each area of a radiative enclosure has a

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specific radiating surface temperature. However, some thermal barrier coating materials that are in common use, such as zirconia,[1–3] are partially transparent to thermal radiation, and so the exchange process is complicated by radiation penetrating into and coming out of each coating surface. The radiation leaving an enclosure surface then depends on the temperature distribution inside its coating that is unknown and is coupled with the exchange process. Radiation from a surface location of the translucent coating is not governed by a single radiating temperature.

To develop an analytical method for this type of radiative exchange, the important case is considered here of the flow of a hot transparent gas in a symmetric parallel-plate channel with each wall cooled on the outside (Fig. 1). The channel walls are metal or opaque plates at a uniform temperature in the flow direction and they each have the same translucent ceramic coating on the inside of the channel for thermal protection. The coatings on the two walls view each other and exchange radiation. However, because of symmetry the radiative flux leaving the outer surface of each coating (at $x = 0$ in Fig. 1) must equal its incident radiative flux. Hence, the temperatures within the coated walls must be found that satisfy the particular condition of zero-net radiation at the coating surface. This differs from having a coating that is heated by convection from a hot transparent gas but is exposed to cold radiative surroundings. In that case there is radiation to the surroundings but there is negligible radiation received; hence, there is a net radiative flux at the coating surface and it is negative.

Radiative transfer computations in a hot coating include internal emission and radiation transport by absorption, re-emission, and scattering; these mechanisms combine with heat conduction. Combined radiation–conduction calculations can become somewhat complicated when the coating properties depend on the radiation frequency. Zirconia also has high scattering relative to absorption,[1] and so the radiative relations must include scattering. With frequency-dependent properties and large scattering the solution of the exact radiative transfer equations is rather involved, and so it is desirable to use simplified methods for analysis if these will yield good results. Some approximate techniques were applied in Ref. 4 for plane layers of translucent materials with isotropic scattering and results were compared with solutions using the complete radiative transfer equations. It was found that the two-flux method gave accurate results for layers with diffuse boundaries. Diffuse boundaries are a reasonable assumption for thermal barrier coatings because they have a somewhat rough exterior and a granular, porous, or columnar structure.

A two-flux method is used here to obtain temperature distributions in translucent thermal barrier coatings on the inside of opaque walls of a parallel-plate channel. The coatings are heated by convection of a hot nonradiating gas in the channel and each wall is facing an identical coated wall with the same heating conditions. There are no temperature variations in the axial direction. Following the ideas in Refs. 5 and 6, a Green's function is derived to provide an analytical solution for the differential equation governing the radiative flux function in the two-flux method. The analytical relations include the special condition for the translucent layer exchanging energy with another identical translucent layer. The Green's function is incorporated into an iterative solution for the temperature distribution and heat flow in the translucent layer and its opaque substrate. It is shown how the method can be applied for a few spectral bands, such as for a material like zirconia that has a frequency region where it is translucent, and is usually assumed opaque for the remainder of the spectrum.

To demonstrate some characteristics of the present results, comparisons are made with the well-known temperature distribution for a gray layer between two parallel black walls that have specified temperatures. The two-flux solution is compared with results from the literature[7] obtained from the exact equations of radiative transfer. The solution for the present boundary condition is then contrasted with the results for a translucent layer between two black boundaries. Results are also included where one side of a translucent layer is heated by convection and is simultaneously exposed to cold radiative surroundings. The solution in the literature[7] is for a translucent layer with a refractive index of 1; additional results are given here for a refractive index of 2.

After considering this basic case where surface temperatures are specified, typical temperature distributions are given for a translucent coating on an opaque substrate with convection on both outer boundaries. An example is also given for temperatures in a thermal barrier coating on a turbine blade that is in a blade row away from the combustor so that it is surrounded by other similarly coated and cooled blades. The blade has zero-net radiation exchange with adjacent coated blades, which is the condition in the present analysis.

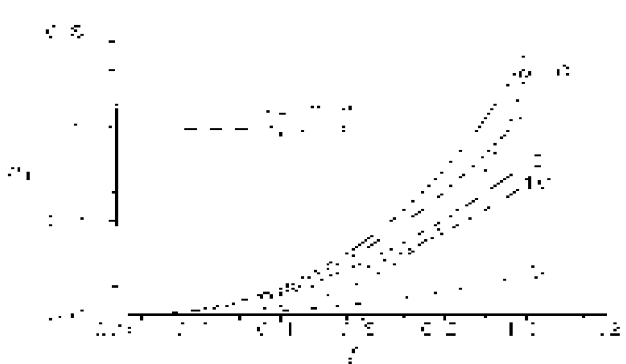

Fig. 1 Geometry and nomenclature for a translucent thermal barrier coating on each opaque wall of a symmetric channel with radiation exchange and convection at exposed boundaries.

Analysis

A parallel-plate channel (Fig. 1) has its opaque walls of thickness δ_m coated on the inside with thermal barrier coatings of thickness δ_c that are translucent for thermal radiation at some frequencies. The x coordinate starts at the inside surface of a coated wall as shown on the right side of Fig. 1. Inside the channel there is flow of a hot transparent gas that convectively heats the walls with a heat transfer coefficient h_1 at the inner boundaries of the thermal barrier coatings. The exterior surfaces of the walls are cooled by convection and radiation; all of the conditions are symmetric about the center of the channel. In the translucent coating the total heat flux is the sum of radiation and conduction, and the energy equation in the coating is given by[3]

$$q_{\text{tot}} = -k_c \frac{\mathrm{d}T}{\mathrm{d}x} + \int_{\nu=0}^{\infty} q_{\nu r}(x)\,\mathrm{d}\nu \tag{1}$$

For steady-state conditions the total heat flux q_{tot} by conduction and radiation is independent of x. To solve this equation for $T(x)$ a relation is needed for the local spectral radiative flux $q_{\nu r}(x)$. In the two-flux method the $q_{\nu r}(x)$ is related to a flux quantity $G_\nu(x)$ by[3]

$$q_{\nu r}(x)\,\mathrm{d}\nu = -\frac{1}{3(a_\nu + \sigma_{s\nu})} \left.\frac{\mathrm{d}G_\nu}{\mathrm{d}x}\right|_x \mathrm{d}\nu \tag{2}$$

where $G_\nu(x)$ is obtained by solving an auxiliary equation that will be defined.

After substituting Eq. (2) into Eq. (1) the result is integrated to give a relation for the temperatures in the coating in terms of some quantities that will be determined

$$T(x) = T(0) - \frac{1}{k_c}\left[q_{\text{tot}}x - \frac{1}{3}\int_{\nu=0}^{\infty} \frac{G_\nu(0) - G_\nu(x)}{a_\nu + \sigma_{s\nu}}\,\mathrm{d}\nu\right] \tag{3}$$

To evaluate the $T(x)$ in the translucent layer the $T(0)$, q_{tot}, and $G_\nu(x)$ must be obtained.

From the two-flux method the flux quantity $G_\nu(x)$ is found by solving a differential equation that includes the local blackbody emission and the scattering albedo[3]

$$\frac{d^2G_\nu(x)}{dx^2} - 3(a_\nu + \sigma_{s\nu})^2(1 - \Omega_\nu)G_\nu(x) = -3(a_\nu + \sigma_{s\nu})^2(1 - \Omega_\nu)4n^2e_{\nu b}(x) \tag{4}$$

The $e_{\nu b}(x)$ will be evaluated by iteration because it depends on the unknown temperature distribution.

To solve the second-order equation [Eq. (4)], two boundary conditions are required. The general boundary condition for $G_\nu(0)$ at the translucent boundary of a coating, $x = 0$, when it is subjected to an incident external radiative flux $q_{\nu r1}$, is[3]

$$G_\nu(0) - \frac{2}{3(a_\nu + \sigma_{s\nu})}\frac{1 + \rho_i}{1 - \rho_i}\left.\frac{dG_\nu}{dx}\right|_{x=0} = 4\,\frac{1 - \rho_o}{1 - \rho_i}\,q_{\nu r1} \tag{5a}$$

where the diffuse reflectivities ρ_i and ρ_o are shown in Fig. 1. From the symmetry of the parallel-plate channel and heating conditions in Fig. 1, the radiative energy entering the boundary of the translucent layer at $x = 0$ must equal the energy leaving. Hence, the net radiative flux at the boundary is zero. Using the condition $q_{\nu r}(0) = 0$ in Eq. (2) gives $dG_\nu/dx|_{x=0} = 0$, so that the general boundary condition Eq. (5a) reduces to

$$G_\nu(0) = 4[(1 - \rho_o)/(1 - \rho_i)]q_{\nu r1} \tag{5b}$$

The incident spectral radiation flux $q_{\nu r1}$ in Eq. (5b) is unknown as it arises from radiation leaving the opposite coated wall, and it depends on the unknown temperature distribution in the coating that is to be obtained by the solution.

The second boundary condition for solving Eq. (4) is at the coating–substrate interface, $x = \delta_c$ and this is given by[3]

$$G_\nu(\delta_c) + \frac{2}{3(a_\nu + \sigma_{s\nu})}\frac{2 - \varepsilon_{m1}}{\varepsilon_{m1}}\left.\frac{dG_\nu}{dx}\right|_{x=\delta_c} = 4n^2e_{\nu b}(\delta_c) \tag{6}$$

Equation (4) with the boundary conditions Eqs. (5b) and (6) can be solved in a convenient form by obtaining a Green's function. Following the detailed procedure in Ref. 5 the Green's function was derived in the following dimensionless form with $X = x/\delta_c$:

$$g_\nu(X,\xi) = \begin{cases} \dfrac{\sinh m_\nu(1-\xi) + P_{m\nu}m_\nu\cosh m_\nu(1-\xi)}{m_\nu(\sinh m_\nu + P_{m\nu}m_\nu\cosh m_\nu)}\sinh m_\nu X, & 0 \le X < \xi \\[2ex] \dfrac{\sinh m_\nu\xi}{m_\nu}\,\dfrac{\sinh m_\nu(1-X) + P_{m\nu}m_\nu\cosh m_\nu(1-X)}{\sinh m_\nu + P_{m\nu}m_\nu\cosh m_\nu}, & \xi < X \le 1 \end{cases} \tag{7}$$

The $g_\nu(X,\xi)$ in Eq. (7) is used to obtain the effect of the nonhomogeneous term in Eq. (4) when computing the dimensionless $\tilde{G}_\nu(X)$. To obtain the complete solution for $\tilde{G}_\nu(X)$ the solution for the homogeneous part of Eq. (4) is also needed. Following the procedure in Ref. 5 the general homogeneous solution is

$$\tilde{G}_{h\nu}(X) = A_\nu \sinh m_\nu X + B_\nu \cosh m_\nu X \tag{8a}$$

The boundary conditions Eqs. (5b) and (6) are applied to give relations for B_ν and A_ν

$$B_\nu = \tilde{G}_{h\nu}(0) \tag{8b}$$

$$A_\nu = \frac{4n^2\tilde{e}_{\nu b}(1) - B_\nu(\cosh m_\nu + P_{m\nu}m_\nu \sinh m_\nu)}{\sinh m_\nu + P_{m\nu}m_\nu\cosh m_\nu} \tag{8c}$$

The $\tilde{G}_h(0)$ and $\tilde{e}_{\nu b}(1)$ must be determined during the solution. By adding $\tilde{G}_{h\nu}(X)$ and the nonhomogeneous solution obtained with $g_\nu(X)$, the solution of Eq. (4) is

$$\tilde{G}_\nu(X) = \tilde{G}_{h\nu}(X) + 4m_\nu^2n^2\int_0^1 g_\nu(X,\xi)\tilde{e}_{\nu b}(\xi)\,d\xi \tag{9}$$

From the form of the Green's function in Eq. (7), the $\tilde{G}_{h\nu}(0) = \tilde{G}_\nu(0)$.

Now that a relation for $\tilde{G}_\nu(X)$ has been developed, relations will be found for $T(0)$ and q_{tot} in Eq. (3). These are obtained by writing total energy flux relations for a coating and its opaque substrate. At the exposed surface of the coating inside the channel the heat flux is only by convection because the net radiative contribution is zero as previously discussed; then at $x = 0$

$$q_{tot} = h_1[T_{g1} - T(0)] \tag{10a}$$

In the translucent coating, by use of Eq. (3) at $x = \delta_c$

$$q_{tot} = \frac{k_c}{\delta_c}[T(0) - T(\delta_c)] + \frac{1}{3}\int_{\nu=0}^{\infty}\frac{G_\nu(0) - G_\nu(\delta_c)}{a_\nu + \sigma_{s\nu}}\,d\nu \tag{10b}$$

At the interface of the semitransparent coating and the opaque substrate, continuity of the temperature gives $T_{coating}(\delta_c) = T_{substrate}(\delta_c)$. For the opaque substrate, heat is transferred only by conduction

$$q_{tot} = (k_m/\delta_m)[T(\delta_c) - T(\delta_c + \delta_m)] \tag{10c}$$

At the cooled side of the substrate heat flow is by convection to the external cooling gas and there is radiative exchange that is assumed here to be to a large surrounding environment with an effective blackbody temperature T_{s2}. Then

$$q_{tot} = h_2[T(\delta_c + \delta_m) - T_{g2}] + \varepsilon_{m2}\sigma[T^4(\delta_c + \delta_m) - T_{s2}^4] \tag{10d}$$

Equations (10a–10d) are solved numerically for $T(0)$, $T(\delta_c)$, $T(\delta_c + \delta_m)$, and q_{tot}. The temperature distribution is then evaluated from Eq. (3) using $G_\nu(X)$ from Eq. (9). An iterative method using dimensionless equations is used to obtain the converged solution for $T(x)/T_{g1}$ as will be described.

A way for determining the $\tilde{G}_{h\nu}(0)$ in Eq. (8b) must be provided. The $q_{\nu r}(0)$ must be zero and this gives from Eq. (4) combined with Eq. (2) and then integrated with respect to x from 0 to δ_c, that at the coating–substrate interface

$$q_{\nu r}(\delta_c) = (a_\nu + \sigma_{s\nu})(1 - \Omega_\nu)\int_{x=0}^{\delta_c}[4n^2e_{\nu b}(x) - G_\nu(x)]\,dx \tag{11a}$$

From the boundary condition at $x = \delta_c$ [Eq. (6)], combined with Eq. (2), there is also the relation

$$q_{\nu r}(\delta_c) = \frac{1}{2}\frac{\varepsilon_{m1}}{2 - \varepsilon_{m1}}[G_\nu(\delta_c) - 4n^2e_{\nu b}(\delta_c)] \tag{11b}$$

As described in the iterative solution procedure, Eqs. (11a) and (11b) must yield the same result when the correct solution is found. Because Eq. (11a) is based on $q_{\nu r}(0)$ being zero, equality of Eqs. (11a) and (11b) will assure that this boundary condition is satisfied. From Eq. (2) this will also yield the condition $\mathrm{d}G_\nu/\mathrm{d}x|_{x=0} = 0$ and this can be used as a numerical check on the converged solution for $G_\nu(x)$.

Multiple Spectral Band Form of the Radiative Relations

Thermal barrier materials such as zirconia have spectral properties with a frequency region that has some transparency, while in the remainder of the spectrum there is very little transmission and the coating is usually assumed opaque. Multiple spectral bands for zirconia or other materials can be used to include spectral property variations in the semitransparent region. The equations are given here for J spectral bands, although for computational simplicity and to still yield reasonable accuracy only a few bands are usually used.

To obtain the temperature distribution in the thermal barrier coating, Eq. (3) is summed over the bands ($j = 1 \ldots J$) in the semitransparent region to give in dimensionless form

$$t(X) = t(0) - \frac{1}{N_c}\left[\tilde{q}_{\mathrm{tot}}X - \frac{1}{3}\sum_{j=1}^{J}\frac{\tilde{G}_j(0) - \tilde{G}_j(X)}{\kappa_j}\right] \qquad (12)$$

To obtain the $\tilde{G}_j(X)$, Eq. (9) is used for each band in the form

$$\tilde{G}_j(X) = \tilde{G}_{hj}(X) + 4m_j^2n^2 \times \int_0^1 g_j(X, \xi)t(\xi)^4\{F_{uj}[t(\xi)] - F_{lj}[t(\xi)]\}\,\mathrm{d}\xi \qquad (13)$$

where $\tilde{G}_{hj}$ and g_j are evaluated using the properties in the jth band. The $F_{uj}(t)$ is the fraction of blackbody energy for emission at t in the frequency range $\nu = 0$ to ν_{uj} that is at the upper limit of the jth band, and F_{lj} corresponds to the lower limit ν_{lj} of the band; $F(t)$ was evaluated from the summation form in Ref. 8. From Eq. (10) the relations for $t(0)$ and $\tilde{q}_{\mathrm{tot}}$ are as follows:

$$\tilde{q}_{\mathrm{tot}} - H_1[1 - t(0)] = 0 \qquad (14a)$$

$$\tilde{q}_{\mathrm{tot}} - N_c[t(0) - t(1)] - \frac{1}{3}\sum_{j=1}^{J}\frac{\tilde{G}_j(0) - \tilde{G}_j(1)}{\kappa_j} = 0 \qquad (14b)$$

$$\tilde{q}_{\mathrm{tot}} - N_w[t(1) - t(1 + \delta_m/\delta_c)] = 0 \qquad (14c)$$

$$\tilde{q}_{\mathrm{tot}} - H_2[t(1 + \delta_m/\delta_c) - t_{g2}] - \varepsilon_{m2}[t^4(1 + \delta_m/\delta_c) - t_{s2}^4] = 0 \qquad (14d)$$

Solution Method

An iterative solution of equations in dimensionless form was used to obtain $t(X)$, and it provided good convergence for all calculations that were made. For the parameters used the $t(X)$ values are in the 0.5–1 range and less than 50 iterations were usually required for $t(X)$ to converge to 5 decimal places. The opaque heat conduction solution obtained by solving Eqs. (14) without the $\tilde{G}$ terms was used as a first guess for $t(X)$. A trial value for $\tilde{G}_{h\nu}(0)$ was assumed, and because the results here are for either a gray coating or a coating with one translucent spectral band, only one value of $\tilde{G}_{h\nu}(0)$ needed to be selected. Then using the radiation properties for each band or constant properties for a gray solution, the Green's function was evaluated for each band from Eq. (7) and the homogeneous solution from Eq. (8), where $\tilde{e}_{\nu b}(1)$ was evaluated using the trial $t(X)$. Using Eq. (13) then gave $\tilde{G}_j(X)$ for each band. Equations (14a–14d) were then solved for $t(0)$ and $\tilde{q}_{\mathrm{tot}}$, and Eq. (12) was used to evaluate a new $t(X)$. A damping factor of 0.05 or 0.1 was usually used to decrease changes in $t(X)$ between successive iterations; this was required for stability. This gave the $t(X)$ to start the next iteration using Eqs. (8) and (13). The two values of $\tilde{q}_{\nu r}(X = 1)$ were calculated for each band from Eqs. (11a) and (11b) in dimensionless form and adjustments were made in $\tilde{G}_{h\nu}(0)$ to move the $\tilde{q}_{\nu r}(1)$ toward equality. The procedure was continued until $t(X)$ converged and the values of $\tilde{q}_{\nu r}(1)$ from Eqs. (11a) and (11b) differed by less than 10^{-5} for the dimensionless flux. The condition $\mathrm{d}\tilde{G}_\nu/\mathrm{d}X|_{X=0} = 0$ was also checked numerically using the final $\tilde{G}_\nu(X)$ results.

Results and Discussion

Some results will be given to illustrate the thermal behavior of translucent coatings on opaque walls of a symmetric parallel-plate channel carrying a convecting high-temperature transparent gas. First, however, a comparison is made with a fundamental solution in the literature. This demonstrates the accuracy of the two-flux method, and by considering three different radiative conditions at $x = 0$ it illustrates the effect of the radiative boundary condition at the exposed surface of the convectively heated translucent coating.

Comparisons with a Fundamental Solution

The results in Fig. 2 are for a gray layer of thickness δ_c with the layer surfaces each maintained at a specified temperature as given in the figure caption. For all results the layer is in contact at $x = \delta_c$ with a black wall that has a temperature (833.3 K), one-half that for the boundary at $x = 0$ (1666.7 K). For the limiting case of an opaque layer the temperature distribution is the straight solid line for heat conduction between the two boundaries at specified temperatures. Reference 7 provides results for temperature distributions by combined radiation and conduction when a translucent layer is between two black boundaries and the layer is gray with a refractive index of $n = 1$. For these conditions there is blackbody radiation into the layer from the boundary at $x = 0$, so that $q_r(0) > 0$. These results, shown as a short dash line in Fig. 2, were obtained using the exact equations of radiative transfer. The radiation absorbed in the translucent layer yields temperatures above those for only heat conduction. The long dash line shows a comparison with results for the same conditions using the two-flux method. The two-flux results were obtained by using a Green's function from Ref. 9 that applies for these boundary

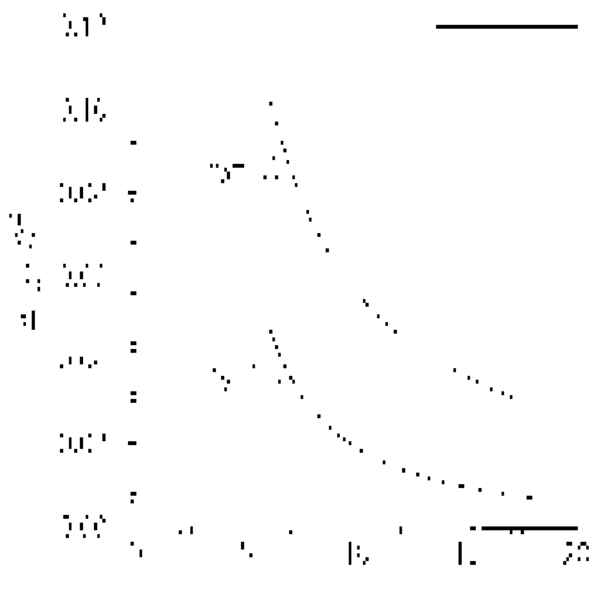

Fig. 2 Comparison of a fundamental solution for a gray translucent layer between two black opaque walls with results for a layer with zero-net radiative flux or zero-incident flux at one boundary. Parameters: $T(x = 0) = 1666.7$ K, $T(x = \delta_c) = 833.3$ K, conduction–radiation parameter $= ka/4\sigma T(0)^3 = 0.02916$ (a = layer absorption coefficient), and refractive indices $n = 1$ and 2.

conditions and is different from the Green's function developed here. As shown in Fig. 2 the temperatures from the two-flux solution are in good agreement with Ref. 7 that is for an absorbing material ($\Omega = 0$) with $n = 1$. Also shown from the two-flux solution is the temperature distribution (dot–dash line) for $n = 2$ that is a typical n for some ceramics. The increased refractive index produces a somewhat more uniform temperature distribution in the central portion of the layer with larger temperature gradients near the boundaries.

To consider the radiative condition being examined here for a parallel-plate channel the two temperature distributions in Fig. 2 labeled identical facing layer are for a symmetric channel geometry and thermal conditions as in Fig. 1. At the coating–substrate interface $x = \delta_c$ there is still a black boundary with its temperature specified (833.3 K) as in Ref. 7, but the boundaries of the coatings inside the channel at $x = 0$ do not have bounding walls. These exposed boundaries are being heated by convection so that they are maintained at the same specified surface temperature (1666.7 K) as in Ref. 7, while having zero-net radiative exchange $q_r(0) = 0$ with the opposite wall coated with an identical translucent layer. The convecting gas inside the channel is nonradiating. For these conditions the temperatures within the translucent layer are decreased relative to those in Ref. 7, where there is a black boundary at $x = 0$ that radiates into the layer. The temperature distributions for $q_r(0) = 0$ have a different curvature as a result of not having net radiation into the coating at $x = 0$. Results are given for refractive indices, $n = 1$ (long dash line) and $n = 2$ (dot–dash line) (Fig. 2); for $n = 2$ the temperature distribution is somewhat more uniform in the central portion of the translucent coating.

For the lowest two curves in Fig. 2 a translucent coating is again bounded by a black wall at $x = \delta_c$ and has its unbounded surface at $x = 0$ heated by convection so that both surface temperatures are the same as in Ref. 7 and for the other results (1666.7 and 833.3 K). The surface at $x = 0$ is now also exposed to cold surroundings so that it has no incident radiation, $q_{r1} = 0$. The hot translucent layer then radiates through its surface to the cold surroundings so that there is a loss in radiative flux at $x = 0$, $q_r(0) < 0$. This decreases the temperatures near $x = 0$ relative to the values for a coating facing a symmetric wall for which $q_r(0) = 0$. The two curves for $q_r(0) < 0$ were obtained by using the analysis in Ref. 6 where the incident radiation flux at $x = 0$ can be specified for a translucent layer.

Typical Temperature Distributions and Heat Fluxes in a Coated Channel Wall

After comparing with the fundamental results in Fig. 2 and demonstrating the effect of various radiative conditions at $x = 0$, results are now considered for coated channel walls as in Fig. 1, where the surface temperature of the translucent coating is not specified. For the results in Figs. 3 and 4, the substrates are opaque with a thermal conductivity independent of temperature and the coatings are translucent and gray. The exposed boundaries of the coatings inside the channel ($x = 0$) and the external boundaries of the opaque substrates ($x = \delta_c + \delta_m$) have both radiation and convection, and so their temperatures must be determined. Before considering the results for symmetric conditions in Fig. 3b, results used for comparison with a different condition are in Fig. 3a. These are for a single coated substrate with the translucent coating exposed to a radiative flux from blackbody surroundings at the temperature of the convecting gas, $q_{r1} = \sigma T_{g1}^4$, and with all other conditions the same as for Fig. 3b; these results were obtained using the analysis in Ref. 6. The substrate thermal conductivity parameter N_w is 10 times that for the coating and results are shown for $N_c = 0.1$ and 1; for convenience in plotting the δ_m is shown as equal to δ_c. The convection coefficients, metal emissivity, and other parameters are in the figure caption. The solid lines are for a coating that is opaque with only heat conduction, but there is radiative exchange at its exposed surface, $x = 0$. The

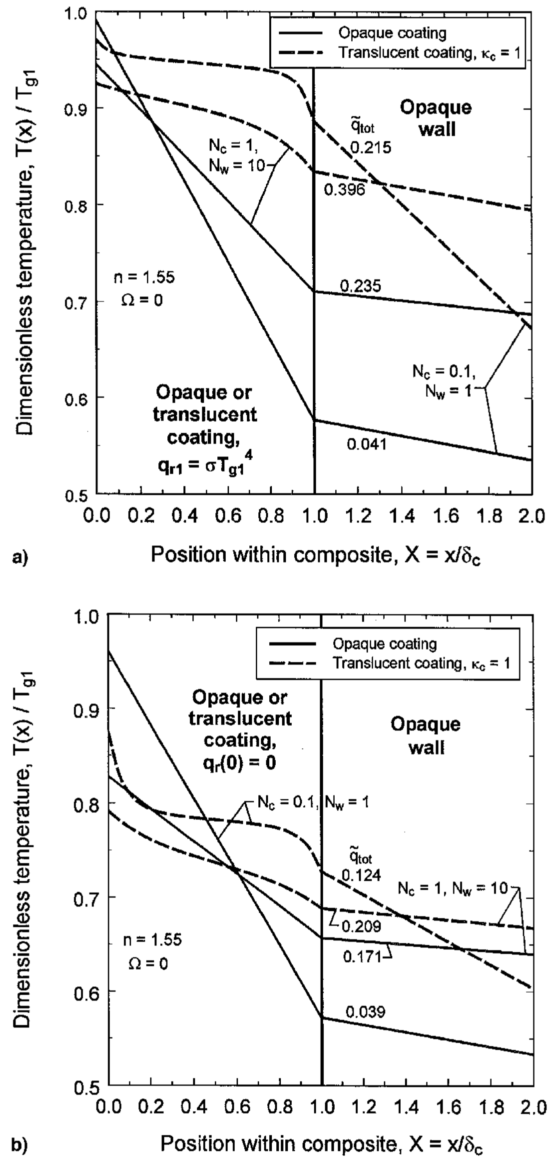

Fig. 3 Effect of thermal conductivity parameters on temperature distributions in a gray translucent coating on an opaque substrate. Parameters: $t_{g2} = t_{s2} = 0.5$, $H_1 = H_2 = 1$, $n = 1.55$, $\Omega = 0$, and $\varepsilon_{m1} = \varepsilon_{m2} = 0.3$. Coating surface at $x = 0$: a) exposed to incident blackbody radiation $q_{r1} = \sigma T_{g1}^4$ and b) zero-net radiation flux $q_r(0) = 0$ as in a symmetric channel.

reflectivity of the coating surface was obtained by using its refractive index in the Fresnel relations[8] and assuming the surface was diffuse. The refractive index is $n = 1.55$, which is typical of glassy materials. For the parameters chosen in Fig. 3a, there is a considerable radiation effect in increasing the coating temperatures, as is evident by comparing temperatures for a translucent coating (dashed lines) with those for an opaque coating (solid lines). The coating temperature distributions have an S shape typical of radiation and conduction in translucent layers. As expected there is less curvature when the coating thermal conductivity contained in the N_c parameter is increased, thereby diminishing the relative radiation effect. Compared with an opaque coating the total heat flow is significantly increased when the coating is translucent. This arises from incident radiation passing through the coating that is moderately transmitting for an optical thickness of $\kappa_c = 1$. For these results the coatings are too translucent to provide good thermal protection for the opaque substrates.

The present analysis was developed to obtain the temperature distribution and heat flux in a coated wall that is exchang-

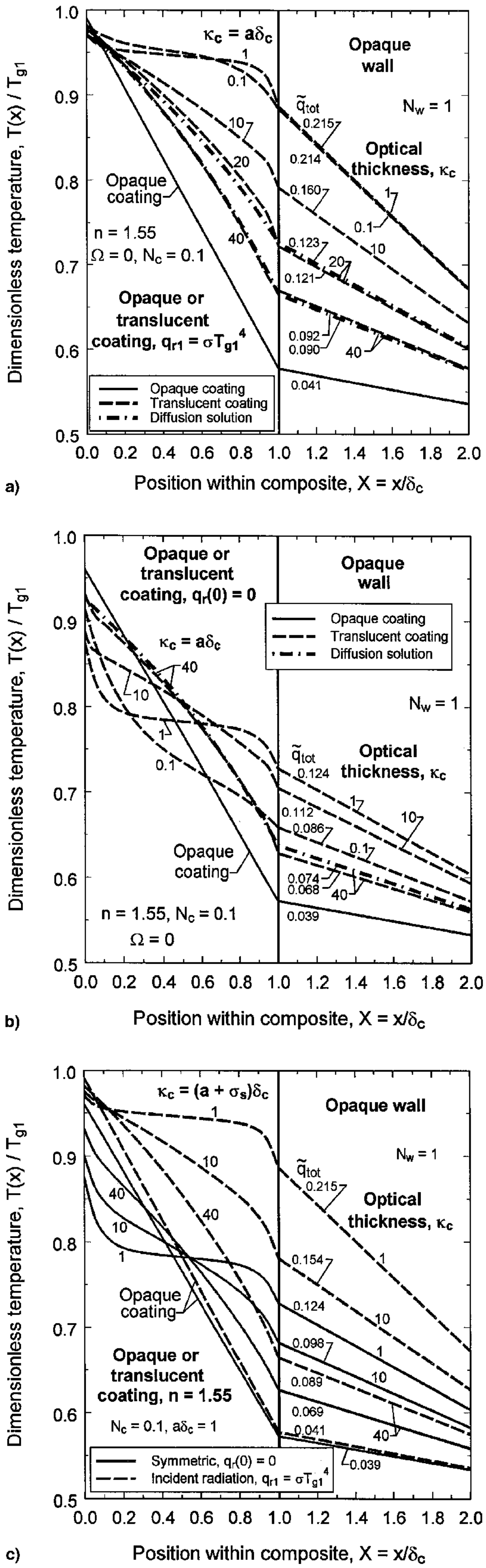

Fig. 4 Effect on coating and wall temperature distributions of the optical thickness of a gray translucent coating on an opaque wall. Parameters: $t_{g2} = t_{s2} = 0.5$, $H_1 = H_2 = 1$, $n = 1.55$, $N_c = 0.1$, $N_w = 1$, $\Omega = 0$, and $\varepsilon_{w1} = \varepsilon_{w2} = 0.3$: a) coating surface at $x = 0$ has incident blackbody radiation $q_{r1} = \sigma T_{g1}^4$ with $\kappa_c = a\delta_c$; b) coating surface at $x = 0$ has zero-net radiation flux, $q_r(0) = 0$, with $\kappa_c = a\delta_c$; and c) effect of scattering: $a\delta_c = 1$ and $\sigma_s\delta_c = 0.9, 9, 39$, so that $\kappa_c = (a + \sigma_s)\delta_c = 1, 10$, and 40.

ing radiation with another identical coated wall in a channel so that $q_r(0) = 0$. By comparing Fig. 3b with Fig. 3a (that is for a coated wall subjected to blackbody surroundings at T_{g1}) the effect of the symmetric radiative exchange condition is found to significantly decrease the coating temperatures as a result of $q_r(0)$ being reduced to zero. The thermal protection by the coatings is much better than for Fig. 3a. Comparing opaque and translucent results in Fig. 3b shows that translucence has less effect on increasing the total heat flux than it had in Fig. 3a, where there was incident radiation. The results in Fig. 3b illustrate the application of the analysis developed here for a symmetric channel and demonstrate typical temperature and heat flux behavior in symmetric channel walls with thermal protection coatings.

The three parts of Fig. 4 are for the same boundary conditions as in Fig. 3. For Fig. 4a a single coated substrate is subjected to blackbody surroundings so there is incident radiation $q_{r1} = \sigma T_{g1}^4$ at $x = 0$. For Fig. 4b the coated walls form a symmetric channel so that $q_r(x) = 0$ at $x = 0$ as treated by the present analysis. Results for both radiative boundary conditions are in Fig. 4c, which shows the effect of scattering. Figure 4 provides additional illustrative results on how the symmetric radiative environment affects temperature distributions and total heat fluxes for various optical thicknesses of the translucent layer.

In Fig. 4a there is no scattering and the coating optical thicknesses are $\kappa_c = a\delta_c = 0.1$–40; the solid lines give the opaque limit. Except for small $a\delta_c$ the results move toward the opaque limit with increasing optical thickness, and the total heat flux decreases as the transmitted radiation is thereby reduced and thermal protection of the substrate is improved. For an optical thickness of $a\delta_c = 40$ there is still a significant internal radiation effect. The dot–dash lines are from the diffusion solution using the relations in the Appendix.[4] For optical thicknesses of 20 or more the diffusion solution provides reasonable engineering accuracy.

In Fig. 4b there is a considerable change in the temperatures when the coating is in a symmetric channel, and so the radiative contribution to the total heat flux is reduced by having $q_r(0) = 0$. These results are without scattering. For small optical thicknesses, $\kappa_c = a\delta_c$, the lack of net radiative flux at $x = 0$ results in a rapid temperature decrease for small x. For increasing optical thickness the temperatures move toward the opaque limit in a different manner than in Fig. 4a.

The results in Figs. 4a and 4b are without scattering and the effect of scattering is now demonstrated. In Fig. 4c the optical thickness for absorption is kept the same, $a\delta_c = 1$, for all of the translucent coatings while the coating optical thickness is increased by adding scattering to give the values shown for $(a + \sigma_s)\delta_c > 1$. The dashed lines are for a coated substrate in black surroundings so that $q_{r1} = \sigma T_{g1}^4$ at $x = 0$; these results can be compared with Fig. 4a. The solid lines are for a symmetric channel and can be compared with Fig. 4b. The relative effect of the two boundary conditions $q_{r1} = \sigma T_{g1}^4$ and $q_r(0) = 0$ on the temperatures and total heat fluxes is similar to that without scattering in Figs. 4a and 4b, where the optical thickness was increased by adding only absorption.

Results for Cooled Turbine Blades

Another application for the present analysis is to examine the effect of translucence for a thermal barrier coating on the outside of a turbine blade when the blade is surrounded by identical cooled blades in a blade row away from radiation incident from the combustor. At the exterior surface of the coating the incident and leaving radiation fluxes are then equal, so that $q_{\nu r}(0) = 0$. In Fig. 5 the blade has a zirconia coating 0.25 mm thick on a 0.762-mm-thick metal wall.[3] From Refs. 1 and 10 the zirconia was specified with a semitransparent spectral region for frequencies $\nu > c_0/(5 \times 10^{-6})\ \mathrm{s}^{-1}$, with absorption and scattering coefficients $a_\nu = 30\ \mathrm{m}^{-1}$ and $\sigma_{s\nu} = 10{,}000\ \mathrm{m}^{-1}$, respectively. The zirconia was assumed opaque

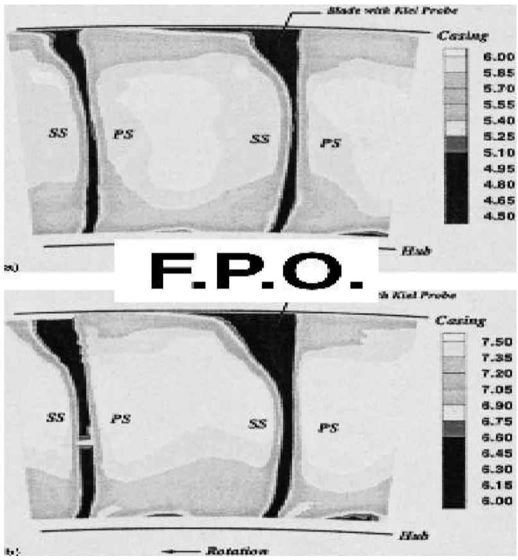

Fig. 5 Temperature distribution for coated turbine blade surrounded by identical cooled blades so that net radiation exchange is zero. Temperatures for clean metal without a coating ($\varepsilon_{m1} = \varepsilon_{m2}$ = 0.3), with an opaque thermal barrier coating, and with a semitransparent coating. Parameters (units are in Nomenclature): h_1 = 3014, h_2 = 3768, k_c = 0.8, k_m = 33, $\delta_c = 0.25 \times 10^{-3}$, $\delta_m = 0.762 \times 10^{-3}$, n = 1.58, a = 30 and $\sigma = 10^4$ for $\nu > c_0/(5 \times 10^{-6})\ \mathrm{s}^{-1}$, T_{g1} = 2000, T_{g2} = 1000, and ε_{m1} = 0.3.

for smaller frequencies. The blade is heated on the outside by convection of a hot gas at 2000 K and is convectively cooled internally with gas at 1000 K. The blade internal cooling passages are sufficiently uniform in temperature that radiative heat exchange is negligible in the passages. The dimensionless convection parameters are in the caption of Fig. 5. The metal surface at the coating–metal interface is assumed to be clean with an emissivity $\varepsilon_{m1} = 0.3$ (larger values such as $\varepsilon_{m1} = 0.6$ can occur if there is a thin bonding material between the coating and the metal). The zirconia exterior surface was assumed diffuse with its reflectivity computed by using the Fresnel optical relations[8] with an index of refraction $n = 1.58$ used for zirconia. The results in Fig. 5 illustrate the radiative effect within the coating for turbine blades away from the combustor radiation, which is the environment for most blades. For this external condition radiation inside the translucent coating has a very small effect on the internal temperatures. A good estimate of internal temperatures is obtained by using the opaque heat-conduction solution. For these conditions, where there is no net radiative exchange at the external surface, heat conduction within the thermal barrier coating is dominating compared with internal radiation for the typical parameters used here, and internal radiation does not degrade the insulating ability of the coating.

Concluding Remarks

A method has been developed to determine wall-temperature distributions for a parallel-plate channel with translucent thermal barrier coatings on its walls and with a high-temperature transparent gas flow. To increase thermal efficiency in heat transfer devices it is desirable to operate at elevated temperatures and it may be necessary to protect structural materials from high-temperature gases. Some coating materials are translucent for thermal radiation, such as zirconia that is in common use. The thermal behavior of translucent coatings on the channel walls is considered here. At elevated temperatures there is radiation exchange between coated surfaces, and the temperature distributions in the translucent coatings and in the metal or other opaque substrates depend on the exchange. However, the exchange depends on the internal temperatures of the coatings that are unknown.

A method of analysis was developed to obtain coating and substrate temperature distributions for the important case of a symmetric parallel-plate channel carrying a high-temperature nonradiating gas. From symmetry the radiation received by a coating inside the channel from the opposite wall is equal to the radiation leaving the translucent coating that is unknown. This radiative condition was incorporated into a method using the two-flux equations that were solved by developing a Green's function. Typical temperature distributions were compared with those for a translucent coating exposed to blackbody surroundings at the gas temperature, and a coating heated by convection in low-temperature surroundings that provide negligible incident radiation so that the coating has a net radiative loss. Comparisons are also made with a diffusion solution that gives good agreement when the coating optical thickness is about 20 or larger. The analytical method provides a means for obtaining radiative effects in a symmetric enclosure, where the walls are not opaque as in the usual enclosure theory, but are each coated with a translucent layer.

An example is also given for a turbine blade protected on the outside with a thermal barrier coating, where the blade is adjacent to identical coated blades and does not receive radiation from the combustor. For this environment, radiation effects in the translucent coating were found to be small relative to conduction for the typical turbine operating conditions selected. Hence, for this environment internal radiation does not degrade the insulating ability of the coating and a heat-conduction analysis provides good results for the temperature distribution.

Appendix: Relations for Diffusion Solution

The equations governing the diffusion solution are similar to those for opaque layers, except with the addition of radiative diffusion terms in the translucent coating. For the calculated results given here the relations were placed in dimensionless form using the parameters in the Nomenclature. For the conditions in Fig. 4b at the exposed boundary $x = 0$ of a gray translucent coating[4]

$$q_{\mathrm{tot}} = h_1[T_{g1} - T(0)] \tag{A1}$$

For the results in Fig. 4a, where $q_r(0) \neq 0$, the following additional radiation term is on the right side: $(1 - \rho_o)[q_{r1} - \sigma T(0)^4]$. For a gray translucent coating with conduction and radiative diffusion

$$q_{\mathrm{tot}} = \frac{k_c}{\delta_c}[T(0) - T(\delta_c)] + \frac{4n^2}{3(a + \sigma_s)\delta_c}\sigma[T(0)^4 - T(\delta_c)^4] \tag{A2}$$

In the opaque substrate heat is transferred only by conduction, so that

$$q_{\mathrm{tot}} = \frac{k_m}{\delta_m}[T(\delta_c) - T(\delta_c + \delta_m)] \tag{A3}$$

At the cooled side of the metal wall heat flow is by convection to the external cooling gas and there is radiative exchange assumed here to be with a large surrounding environment that has an effective blackbody temperature of T_{s2}

$$q_{\mathrm{tot}} = h_2[T(\delta_c + \delta_m) - T_{g2}] + \varepsilon_{m2}\sigma[T(\delta_c + \delta_m)^4 - T_{s2}^4] \tag{A4}$$

Equations (A1–A4) are solved numerically for $T(0)$, $T(\delta_c)$, $T(\delta_c + \delta_m)$, and q_{tot}. By using a root solver the temperature distribution $T(x)$ is then evaluated from

$$q_{\mathrm{tot}} = \frac{k_c}{\delta_c}[T(0) - T(x)] + \frac{4n^2}{3(a + \sigma_s)\delta_c}\sigma[T(0)^4 - T(x)^4] \tag{A5}$$

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